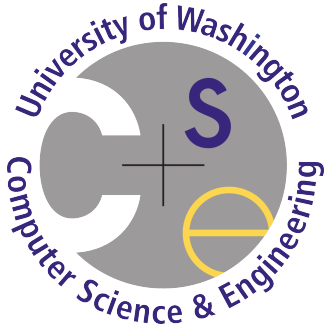
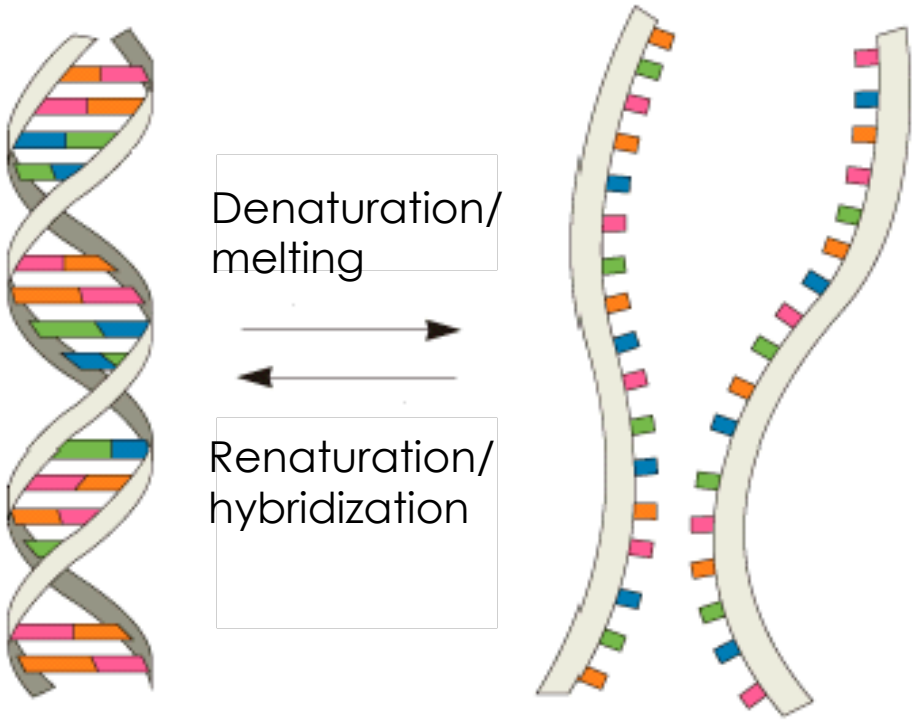


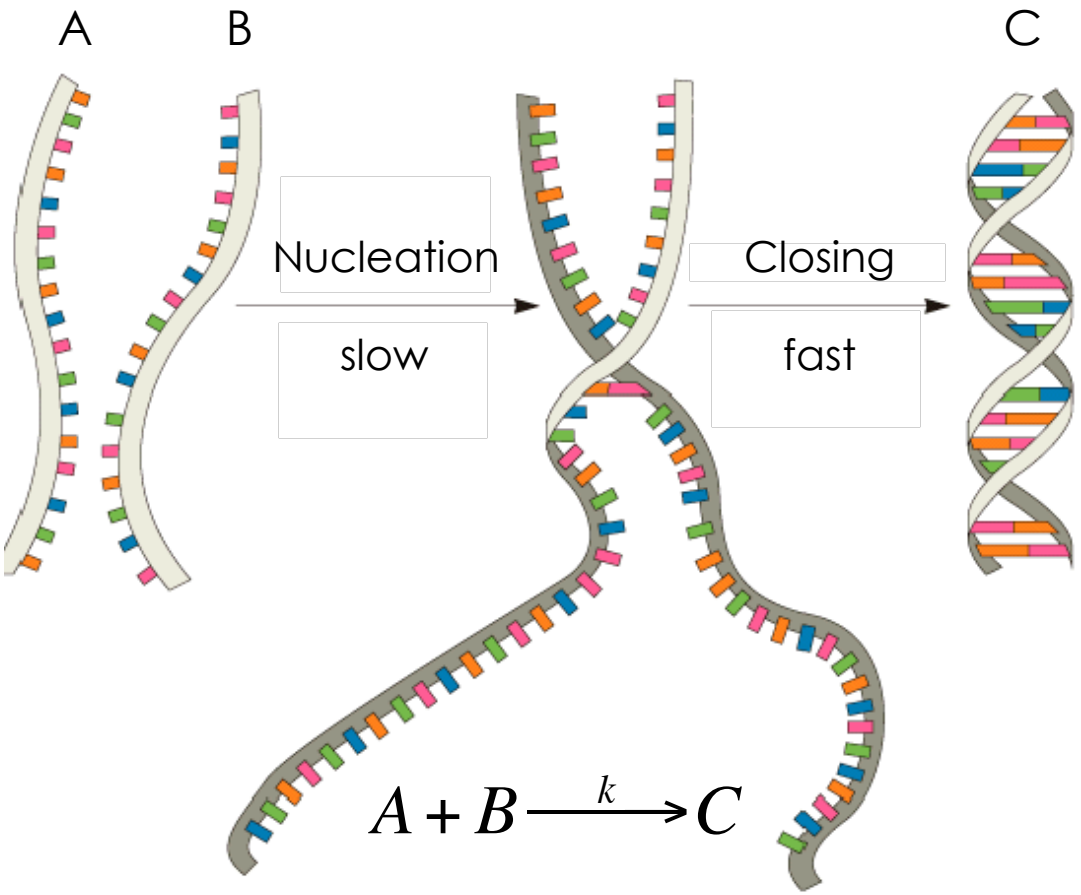
Module 4: Chemical reaction networks (CRNs)

CSE590: Molecular programming and neural computation.

DNA hybridization



DNA hybridization



DNA hybridization is a multi-step process but is well-modeled as a Single bimolecular reaction

Reaction stoichiometry

Bimolecular reaction: $A + B \xrightarrow{k} C$

Q: What do you get when you system starts with 4A and 3B?
A: 1A and 3B

Unimolecular reaction: $A \xrightarrow{k} B + C$

Q: What do you get when you system starts with 4A and 3C?
A: 7C and 4B

Catalytic (bimolecular) reaction: $A + B \xrightarrow{k} A + C$

Q: What do you get when you system starts with 1A and 4B?
A: 1A and 4C

Computing with stoichiometry

Q: What chemical reaction calculates the function f(x)=2x?

$$X \rightarrow 2Y$$

Q: What chemical reaction calculates f(x)=x/2

$$X + X \rightarrow Y$$

David Doty and David Soloveichik (2013)

Computing with stoichiometry

Q: What chemical reaction calculates the function f(x1,x2)=min(x1,x2)?

$$X_1 + X_2 \rightarrow Y$$

Q: What chemical reaction **network** (CRN) calculates the sum of two species?

$$X_1 \xrightarrow{k} Y$$

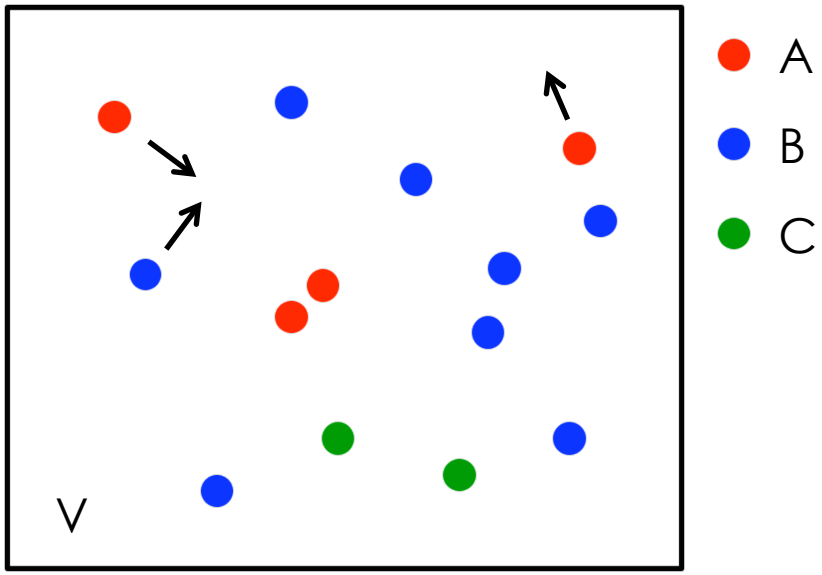
$$X_2 \xrightarrow{k} Y$$

Theorem: f is computable by a CRN if and only if f is continuous and piecewise rational linear.

David Doty and David Soloveichik (2013)

Mass action kinetics

Q: How does the content of our test tube change given a specific set of reactions between the molecules in the test tube?



$$A + B \xrightarrow{k} C$$

Mass action kinetics

Assumptions: the system is **well-mixed** and **memoryless**.

N_A: # of A molecules
N_B: # of B
N_C: # of C

qdt: Probability that one specific pair (A,B) reacts in the time interval dt

$dN_A=-q\ dt\ N_AN_B=dN_B$

Concentration: $[A]=N_A/V$

$dN_A/V=-\ q\ dt\ V\ (N_A/V)\ (N_B/V)$

$d[A]=-k\ [A][B]\ dt \qquad k:=qV$

$d[A]/dt=-k[A][B]$

Mass action kinetics

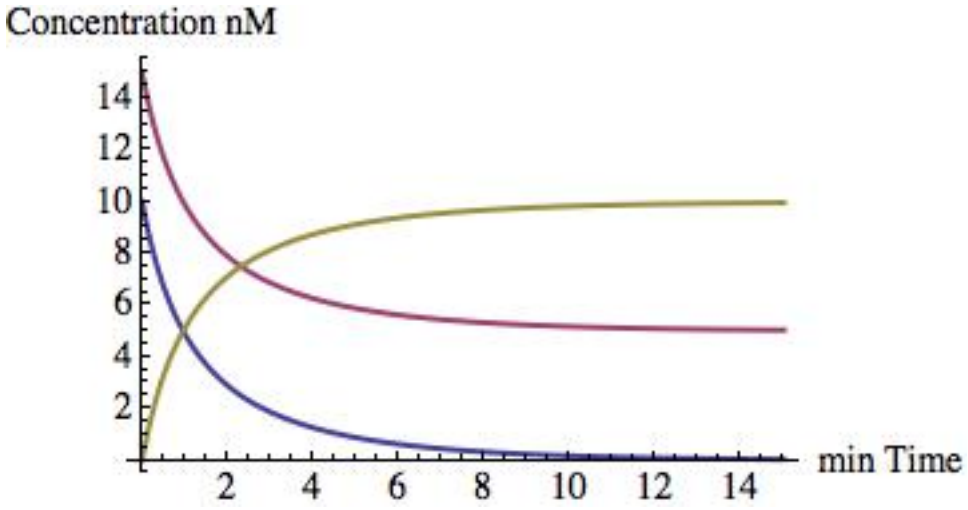
$$A + B \xrightarrow{k} C \Rightarrow \frac{d[A]}{dt} = -k[A][B]$$

$$\frac{d[A]}{dt} = \frac{d[B]}{dt} = -\frac{d[C]}{dt}$$

$$[A](t=0) = A_0$$

$$[B](t=0) = B_0$$

$$[C](t=0) = C_0$$



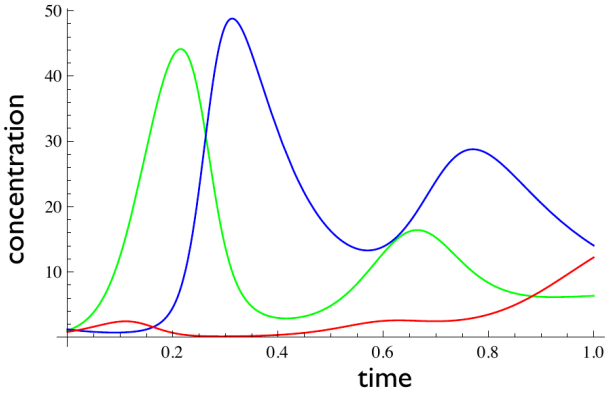
Chemical reaction networks

Syntax:

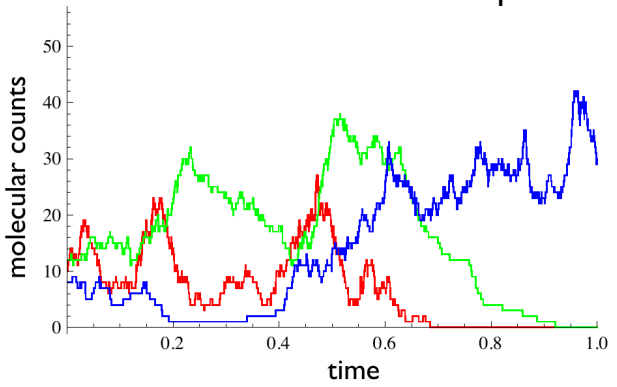
$$
\begin{aligned}
A &\xrightarrow{0.03} 2A \\
2A &\xrightarrow{5\times10^{4}} A \\
B + A &\xrightarrow{10^{5}} 2B \\
B &\xrightarrow{0.01} \\
A + C &\xrightarrow{10^{5}} \\
C &\xrightarrow{0.0165} 2C \\
2C &\xrightarrow{5\times10^{4}} C
\end{aligned}
$$

Semantics:

Mass action (continuous):
Deterministic ODEs



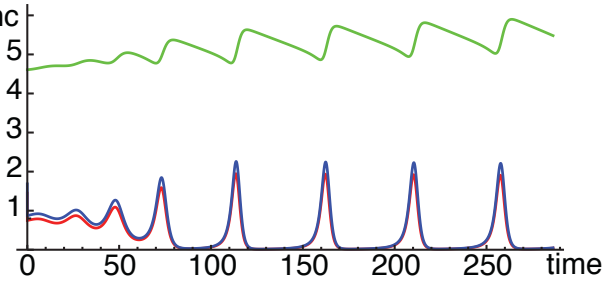
Stochastic (discrete):
Probabilistic Markov process



Chemical reaction networks

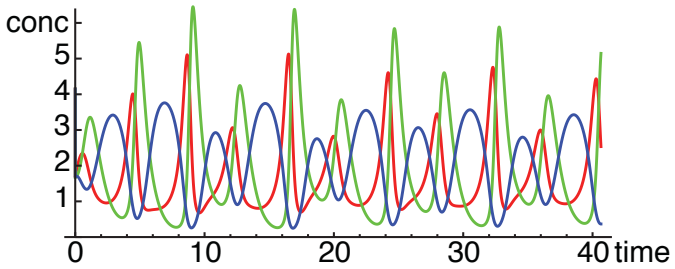
Oregonator (limit cycle oscillator)

$$X_2 \to X_1$$
$$X_2 + X_1 \to \emptyset$$
$$X_1 \to 2X_1 + X_3$$
$$2X_1 \to \emptyset$$
$$X_3 \to X_2$$
$$X_3 \to \emptyset$$



Rössler (chaotic)

$$X_1 \to 2X_1$$
$$2X_1 \to X_1$$
$$X_2 + X_1 \to 2X_2$$
$$X_2 \to \emptyset$$
$$X_1 + X_3 \to \emptyset$$
$$X_3 \to 2X_3$$
$$2X_3 \to X_3$$



The CRN formalism provides a powerful language to reason about Chemical systems but many (mathematically) interersting CRNs have now instantiation in biology or chemistry.

Outlook: Chemical reaction networks

Can we use the CRN formalism as a (prescriptive) programming language?